Linewidth of ferromagnetic resonance for systems with anisotropic damping

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Theory has predicted that the damping of magnetization dynamics may be anisotropic; i.e., it may depend on the momentary orientation $\mathbf{e}(t)$ of the magnetization in the crystal. In the present Brief Report it is shown that in general this anisotropy is averaged out at least to some extent for the special trajectory $\mathbf{e}(t)$ of the magnetization vector in a ferromagnetic-resonance (FMR) experiment. In principle it may be that there is no anisotropy of the FMR linewidth although the damping is strongly anisotropic and although this anisotropy will be essential in a more complicated trajectory $\mathbf{e}(t)$ of the magnetization vector.

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In recent years there has been a great research activity on fast magnetization dynamics for micro- and nanosized magnets because of their potential use in advanced information storage and data processing devices, with special emphasis on domain-wall dynamics (in particular for nanowires[1]), on magnetization reversal in nanomagnets,[1] and on vortex dynamics.[2] Thereby, most theoretical investigations were based on the Gilbert equation of motion[3] for the magnetization $\mathbf{M}(\mathbf{r},t)$,

$$\frac{d\mathbf{M}}{dt} = -\gamma(\mathbf{M}\times\mathbf{H}_{\mathrm{eff}}) + \frac{1}{M}\mathbf{M}\times\alpha\frac{d\mathbf{M}}{dt}. \tag{1}$$

Here, γ is the gyromagnetic ratio; $\mathbf{H}_{\mathrm{eff}}$ is the effective field composed of the external field $\mathbf{H}$, the exchange field $\mathbf{H}_{\mathrm{ex}}$, the anisotropy field $\mathbf{H}_{\mathrm{ani}}$, and the dipolar field $\mathbf{H}_{\mathrm{dip}}$; and α is the scalar damping constant. Equation (1) is the simplest conceivable equation which describes precession of $\mathbf{M}(\mathbf{r},t)$ around $\mathbf{H}_{\mathrm{eff}}$ (first term on the right-hand side) as well as damping (second term). Of course the question arises (see, e.g., Refs. 4–6) as to whether this equation really encompasses all the relevant physics for the time scale of the above discussed experiments, which is the near-adiabatic time scale[4] between several picoseconds and nanoseconds. For instance, Gilbert[3,6] himself suggested a generalization of his equation of motion, Eq. (1), with the local damping term and the scalar constant α replaced by a nonlocal damping term (which relates the dynamics of the magnetization at site $\mathbf{r}$ to the dynamics of the magnetization at all other sites $\mathbf{r}'$) with a damping matrix $\underline{\underline{\boldsymbol{\alpha}}}(\mathbf{r},\mathbf{r}')$. A damping of the dynamics described by a damping matrix was suggested also by Baryakhtar *et al.*[7] (and Safonov[8]), who already mentioned that the replacement of the damping scalar by a damping matrix might have an influence on the FMR linewidth. Finally, such a generalized equation for the atomic magnetic moments $\mathbf{M}_{\mathbf{R}}=M_{\mathbf{R}}\mathbf{e}_{\mathbf{R}}$ at the atomic sites $\mathbf{R}$ with magnitude $M_{\mathbf{R}}$ and orientation $\mathbf{e}_{\mathbf{R}}$ could be derived[9] by a combination of the *ab initio* electron theory with the phenomenological breathing Fermi-surface model of Kambersky[10] for the magnetization dynamics close to the adiabatic limit,

$$\frac{d\mathbf{e}_{\mathbf{R}}}{dt} = -\gamma\mathbf{e}_{\mathbf{R}}\times\mathbf{H}_{\mathrm{eff},\mathbf{R}}(\{\mathbf{e}_{\mathbf{R}''}\}) + \mathbf{e}_{\mathbf{R}}\times\sum_{\mathbf{R}'}\underline{\underline{\boldsymbol{\alpha}}}_{\mathbf{R},\mathbf{R}'}(\{\mathbf{e}_{\mathbf{R}''}\})\cdot\frac{d\mathbf{e}_{\mathbf{R}'}}{dt}. \tag{2}$$

In Eq. (2) the effective field $\mathbf{H}_{\mathrm{eff},\mathbf{R}}$ depends on the magnetic configuration $\{\mathbf{e}_{\mathbf{R}''}\}$ of the whole system. This is not surprising and is accepted by everybody because also the field $\mathbf{H}_{\mathrm{eff}}(\mathbf{r})$ of Gilbert equation (1) depends on the magnetization field $\mathbf{M}(\mathbf{r})$ of the whole system via the exchange field and the dipolar field. Furthermore, the damping matrices $\underline{\underline{\boldsymbol{\alpha}}}_{\mathbf{R},\mathbf{R}'}$ depend on the magnetic configuration of the whole system, which was not anticipated by Gilbert.[6] Within the breathing Fermi-surface model this is a natural consequence of the fact that both quantities $\mathbf{H}_{\mathrm{eff},\mathbf{R}}$ and $\underline{\underline{\boldsymbol{\alpha}}}_{\mathbf{R},\mathbf{R}'}$ are related to the derivatives $\partial\varepsilon_{j\mathbf{k}}/\partial\mathbf{e}_{\mathbf{R}}$ of the single-electron energies (wave vector $\mathbf{k}$, band index j), and these derivatives depend on the orientations $\{\mathbf{e}_{\mathbf{R}''}\}$ of all magnetic moments in the system. For a collinear situation, i.e., $\mathbf{e}_{\mathbf{R}}(t)=\mathbf{e}(t)$ for all $\mathbf{R}$, the $\varepsilon_{j\mathbf{k}}$ depend on the orientation $\mathbf{e}$ via the spin-orbit coupling. In a time-dependent noncollinear situation the $\varepsilon_{j\mathbf{k}}$ change in addition because of the interatomic exchange interactions.

For a collinear situation, Eq. (2) reduces to

$$\frac{d\mathbf{e}}{dt} = -\gamma\mathbf{e}\times\mathbf{H}_{\mathrm{eff}}(\mathbf{e}) + \mathbf{e}\times\underline{\underline{\boldsymbol{\alpha}}}(\mathbf{e})\cdot\frac{d\mathbf{e}}{dt}, \tag{3}$$

which means that the constant damping scalar α of the Gilbert equation is replaced by a damping matrix which depends on the momentary orientation $\mathbf{e}(t)$ of the homogeneous magnetization.[4,9–12] The matrix $\underline{\underline{\boldsymbol{\alpha}}}$ has at most two nonzero eigenvalues α_p, with $p=1,2$, which correspond to two eigenvectors, and each of them describes the momentary damping which we would have if the momentary $d\mathbf{e}/dt$ was parallel to the respective eigenvector. Altogether, we thus have two types of anisotropy contributions, one related to the momentary orientation $\mathbf{e}(t)$ (contribution 1) and the other one related to the momentary change in orientation, $d\mathbf{e}/dt$ (contribution 2). It has been shown by *ab initio* calculations for bulk Fe, Co, and Ni (Refs. 11 and 12) that the two eigenvalues α_p in fact depend on the momentary $\mathbf{e}(t)$. For instance, in Co one of them varies by a factor of more than 3 when changing the direction $\mathbf{e}(t)$. This anisotropy is very strong in systems with

reduced dimensionality such as monatomic layers or monatomic wires where there are orientations for which the damping is identically zero.[12] As an example, Fig. 4(b) of Ref. 12 shows the calculated eigenvalues α_p for a monatomic hexagonal layer of Ni, with zeroes due to symmetry in some high-symmetry directions and along the high-symmetry line from $[10\bar{1}0]$ to $[2\bar{1}\bar{1}0]$. It has been suggested[12] that this anisotropy of the damping represents a further option to optimize the magnetization-reversal process in nanostructures by choosing an appropiate trajectory $\mathbf{e}(t)$.

The range of validity of the breathing Fermi-surface model was discussed most recently by Gilmore *et al.*[13] with calculations based on the theory of Kamberský[14] for the random-phase approximation (RPA) circular susceptibility. This theory is more general than the breathing Fermi-surface model on one hand, but is also more restricted on the other hand because it takes into account only spin-orbit coupling (no noncollinearity), and because it is valid only for a situation where the magnetization is oriented in a high-symmetry direction of the crystal. Gilmore *et al.*[13] showed that under these circumstances the breathing Fermi-surface model is valid if it suffices to take into account only scattering processes between electronic states in the same band, and that this is justified for low temperatures where the damping is proportional to the electronic momentum relaxation time τ and hence to the Drude conductivity σ. At high temperatures the interband transitions become relevant, leading to a damping which is proportional to $1/\tau$ and hence to the resistivity ρ. For Ni these two contributions to damping could be resolved experimentally,[15] whereas for Fe only the high-temperature behavior could be observed in the experimental temperature range.[16]

It should be noted explicitly that all the above discussions and the following discussions hold only for an equation of motion related to the damping contributions of the breathing Fermi-surface model. In reality, there are also other contributions[13] (for a more general discussion see Ref. 17).

Of course the question arises as to whether the anisotropy of the damping matrix $\underline{\underline{\boldsymbol{\alpha}}}(\mathbf{e})$ for a collinear magnetization configuration can be observed directly experimentally, e.g., by studying the anisotropy of the linewidth of the ferromagnetic resonance (FMR). The problem of the FMR linewidth in a system with anisotropic damping was pointed out already in Ref. 9, and in the present Brief Report it will be solved quantitatively. As discussed in the last paragraph, one could expect such an anisotropy for the case of Ni, whereas for Fe the situation is not clear because for this material the breathing Fermi-surface model is not valid. To test for the anisotropy, one should compare the linewidths for two directions of the static FMR bias field which are crystallographically nonequivalent. For instance, for a magnetic film the easy and hard directions are often determined by the demagnetization field (i.e., by the dipolar contributions to $\mathbf{H}_{\text{eff}}$) but are otherwise crystallographically equivalent. In the *ab initio* versions of the breathing Fermi-surface model[4,9,11,12] which yield Eqs. (2) and (3), the effect of a demagnetization field is not included. Therefore these theories cannot make any prediction on whether the damping is different for two directions which differ just in the demagnetization field but which are otherwise crystallographically equivalent, i.e., which correspond to the same spin-orbit coupling energy. Finally, the FMR data for different directions of the FMR bias field should not be analyzed with Gilbert equation (1), but with the modified equation of motion (3). This equation can be cast into a momentary Gilbert-type equation of motion of the form of Eq. (1) with a damping scalar α (rather than a damping matrix $\underline{\underline{\boldsymbol{\alpha}}}$) which—however—is not a constant as in the Gilbert theory but which depends on the momentary $\mathbf{e}(t)$ and $d\mathbf{e}/dt$ and in which the eigenvalues α_p enter in a complicated manner; see Eqs. (48)–(51) of Ref. 12. The problem is that in an FMR experiment the continuous rotation of $\mathbf{e}(t)$ around a fixed orientation prescribed by the static FMR bias field may average out at least in part the anisotropy of the momentary damping scalar α. Therefore it is not clear to what extent the anisotropy of the damping can be investigated by an FMR experiment. It is the purpose of the present Brief Report to investigate this problem quantitatively.

In our model calculation we assume that the bias field $\mathbf{H}=H\mathbf{e}_z$ is much stronger than the anisotropy field and the demagnetization field so that $\mathbf{e}(t)$ rotates around $\mathbf{e}_z$, driven by the circular FMR field $\mathbf{H}_1$ in the x-y plane with frequency ω and strength $H_1 \ll H$. We then calculate the dissipated power P, which is the average of

$$\frac{dW}{dt} = -\mathbf{H}_1 \cdot \frac{d\mathbf{M}}{dt} \tag{4}$$

taken over one period $2\pi/\omega$ with $\frac{d\mathbf{M}}{dt}$ given by $M_s \frac{d\mathbf{e}}{dt}$, where M_s is the absolute value of the magnetic moment of the sample and where $d\mathbf{e}/dt$ is given by Eq. (3). With $\bar{\alpha}=(\alpha_1+\alpha_2)/2$ and $\Delta\alpha=\alpha_1-\alpha_2$, Eq. (4) yields

$$P = -\gamma M_s H_1^2 \frac{\bar{\alpha}}{(1-x)^2+(\bar{\alpha})^2-\frac{(\Delta\alpha)^2}{4}+\frac{(\Delta\alpha)^2 x^2}{(1+x)^2+(\bar{\alpha})^2-\frac{(\Delta\alpha)^2}{4}}} \tag{5}$$

with

$$x = \frac{\gamma H}{\omega}. \tag{6}$$

For $\Delta\alpha=0$, Eq. (5) reduces to the well-known expression from the conventional Gilbert equation, with the Gilbert scalar α replaced by $\bar{\alpha}$,

$$P = -\frac{\gamma M_s H_1^2 \omega^2 \bar{\alpha}}{(\omega-\gamma H)^2+(\bar{\alpha}\omega)^2}, \tag{7}$$

corresponding to the linewidth [half-width at half maximum (HWHM)]

$$\Delta H = \bar{\alpha}\frac{\omega}{\gamma}. \tag{8}$$

Whereas the Gilbert scalar α is assumed to be constant, the mean value $\bar{\alpha}$ may depend on the orientation of $\mathbf{H}$ in the crystal; i.e., ΔH may be anisotropic. This holds, e.g., for the case of Ni (see Fig. 1), where the two eigenvalues are the

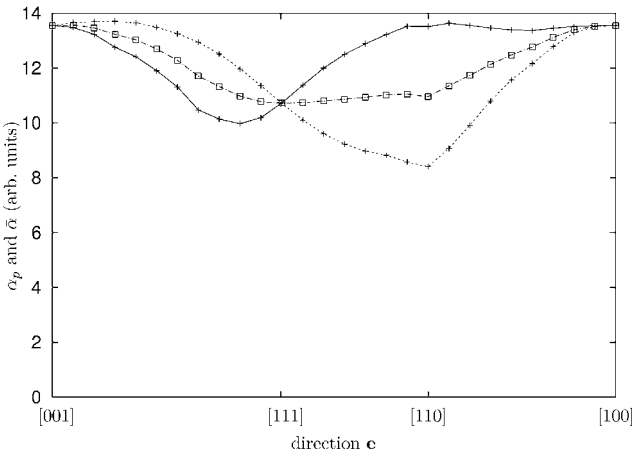

FIG. 1. Anisotropic damping in bcc-Ni, calculated within the breathing Fermi-surface model. The two eigenvalues $\alpha_1(\mathbf{e})$ and $\alpha_2(\mathbf{e})$ of the damping matrix $\underline{\underline{\boldsymbol{\alpha}}}(\mathbf{e})$ are given by the full and dashed lines (symbol +). The mean value $\bar{\alpha}$ is given by the dot-dashed line (symbol □).

same for the orientations [111] and [001], respectively, so that $\Delta\alpha$ is zero, but $\bar{\alpha}$ is different for the two orientations, and thus the anisotropy should show up in the linewidth.

For the more general situation for which $\Delta\alpha$ is nonzero, we analyze Eq. (5) by inserting reasonable values for $\bar{\alpha}$ and $\Delta\alpha$, i.e., $\bar{\alpha}, \Delta\alpha \approx$ some 0.01. Then the denominator in the last term of the denominator of Eq. (5) is dominated by $(1+x)^2$, which is 4 for the case of resonance ($\omega = \gamma H$), and the last two terms in the denominator nearly cancel close to resonance so that Eq. (7) holds approximately also in the general case with $\Delta\alpha \neq 0$. This means that the FMR power absorption P is nearly completely determined by the mean value $\bar{\alpha}$. If $\bar{\alpha}$ is different for two orientations (as for [111] and [001] in Ni; see above) then the linewidth is also different. Indeed, for Ni an anisotropy of the linewidth has been observed for low temperatures.[18] For other materials FMR experiments did not show a clear indication of an anisotropic intrinsic linewidth (see, e.g., Ref. 19), but—as discussed above—it may be that in these materials the preconditions for the applicability of the breathing Fermi-surface model (which predicts an anisotropic intrinsic linewidth) are not fulfilled.

For instance, in the case of Fe, Mosendz *et al.*[20] performed FMR experiments and found a frequency dependence of the FMR linewidth which is well described by a simple Gilbert damping. As the above calculation has shown, this is indeed to be expected even for a system with anisotropic damping. The observed angular dependence of the FMR linewidth in Fe was shown[17] to be mostly caused by extrinsic contributions (two-magnon scattering).

Altogether, we have shown that the effect of the anisotropy contribution 1 on the mean value $\bar{\alpha}$ can be investigated by the anisotropy of the FMR linewidth. The question arises as to whether there are situations for which the existence of two eigenvalues (anisotropy contribution 2) and their dependence on the orientation $\mathbf{e}(t)$ (contribution 1) are relevant. If we want to see the two eigenvalues separately, we have to perform experiments which enforce magnetization trajectories corresponding to just one of these eigenvalues, respectively. For instance, we could investigate a precessional switching by 180° in a very strong external field which rotates the magnetization according to one of the two respective eigenvectors. (Thereby one should take into account that the respective trajectories average over the anisotropy contribution 1, but nevertheless the two averages will be different in general.) Anyway, both anisotropy contributions are physically relevant for dynamics for which the trajectory is not restricted by a simple experimental approach to a situation where the effect of the two eigenvalues and their dependence on $\mathbf{e}(t)$ is necessarily averaged out.

To conclude, we have calculated the width ΔH of the FMR power absorption line for systems with anisotropic magnetization damping. In such systems, the damping is described by a damping matrix $\underline{\underline{\boldsymbol{\alpha}}}(\mathbf{e})$ which exhibits two anisotropic eigenvalues $\alpha_1(\mathbf{e})$ and $\alpha_2(\mathbf{e})$, i.e., eigenvalues which depend on the orientation $\mathbf{e}$ of the magnetization. The linewidth ΔH is nearly completely determined by the mean value $\bar{\alpha}$ of these two eigenvalues. There may be situations for which $\bar{\alpha}$ depends on $\mathbf{e}$, and then ΔH should be anisotropic. However, there may be also various orientations for which $\bar{\alpha}$ is nearly the same, whereas the two eigenvalues are drastically different. Then the FMR experiment does not give any hint of the anisotropy of the damping which, however, may be essential for a trajectory $\mathbf{e}(t)$ which is more complicated than the trajectory for the FMR experiment.

Note added in proof. In the meanwhile we have redone the calculations for a linear rather than a circular FMR field. The numerical results are very similar to those for the circular field, but we did not manage to derive a simple analytical equation for the dissipated power P.

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